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M. Bassetti : NUMERICAL COMPUTATIONS OF SPACE CHARGE EFFECTS IN A POSITRON AND ELECTRON STORAGE RING.

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Appendix

NUMERICAL COMPUTATIONS OF SPACE CHARGE EFFECTS IN A POSITRON AND ELECTRON STORAGE RING.

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In this computations we simulate the interaction effect between two beams with the following approximations:

a) thin lens approximation;

b) vertical effect only, supposing the beams are sheet-like;

c) each beam schematized as having 1000 particles.

The points on the shown diagram (see Fig. 1) are obtained by a computer in the following way:

At first we take 2 groups of 1000 particles on the z, z' plane, whose density follows the law:

$$\rho(z, z') = \rho_o e^{-\frac{z^2 + z'^2}{2\bar{z}^2_\pm}} \qquad [1]$$

where $\bar{z}^2_+$ and $\bar{z}^2_-$ may be given.

To every particle of the two groups is applied a matrix simulating the machine zone between on interaction and the next one

$$\begin{vmatrix} z_{2i} \\ z'_{2i} \end{vmatrix} = \begin{vmatrix} (1-\varepsilon)\cos\mu & (1-\varepsilon)\,\mathrm{sen}\mu \\ -(1-\varepsilon)\,\mathrm{sen}\mu & (1-\varepsilon)\cos\mu \end{vmatrix} \begin{vmatrix} z_{1i} \\ z'_{1i} \end{vmatrix} \qquad [2]$$

Then to every particle is added or substracted (the sign is taken at random) an angular discontinuity $\Delta z'$ as if it had been applied in a preceding sec tion chosen at random (the choice is made among 50 possibilities).

$$\begin{vmatrix} z_{3i} \\ z'_{3i} \end{vmatrix} = \begin{vmatrix} z_{2i} \pm z_n \\ z'_{2i} \pm z'_n \end{vmatrix} \qquad z_n^2 + z'^2_n = \Delta z'^2 \qquad n = 1 \ldots 50 \qquad [3]$$

Let us suppose now that there be no beam-beam interaction. In this case applying many times ($> 1/\varepsilon$) the operations [2] and [3] the two group of particles reach a mean steady distribution with a gaussian distribution which has a mean square width is

$$<\bar{z}^2>^{1/2} = \frac{\Delta z'}{2\sqrt{\varepsilon}}$$

indipendent from initial conditions.

In our calculations we have generally assumed

$$\begin{cases} \Delta z' = 4{,}2 \times 10^{-4} \\ \varepsilon = 6 \times 10^{-3} \end{cases}$$

and we have also considered a case with

$$\begin{cases} \Delta z' = 1{,}715 \times 10^{-4} \\ \varepsilon = 10^{-3} \end{cases}$$

With these assumptions it results

$$<\bar{z}^2>^{1/2} = 2{,}7 \times 10^{-3}$$

After [2] and [3] the program computer the luminosity, defined by

$$L = \int \rho_+ \rho_- dz \simeq \sum_i \frac{\Delta n_i^+}{\Delta z}\frac{\Delta n_i^-}{\Delta z}\Delta z = \sum_i \frac{\Delta n_i^+ \Delta n_i^-}{\Delta z}$$

In the absence of interaction we obtain

$$L_o = \frac{N^2}{2\sqrt{\pi} <\bar{z}^2>^{1/2}} = 1{,}04 \cdot 10^8 \quad \text{where N is } 10^3$$

Let us suppose now that there is interaction. In this case, after [2] [3] and [4], berofe reemploying [2] we calculate the interaction as follows.

For each of the 1000 particles of either group we calculate the difference between the number of particles of the other groups which are above and those which are under the considered particle and we add an angular discontinuity given by

$$z'_{4i} = z'_{3i} + (n_i^+ - n_i^-) \cdot DE_+ \qquad \text{electrons}$$

$$[5]$$

$$z'_{4j} = z'_{3j} + (n_j^+ - n_j^-) \cdot DE_- \qquad \text{positrons}$$

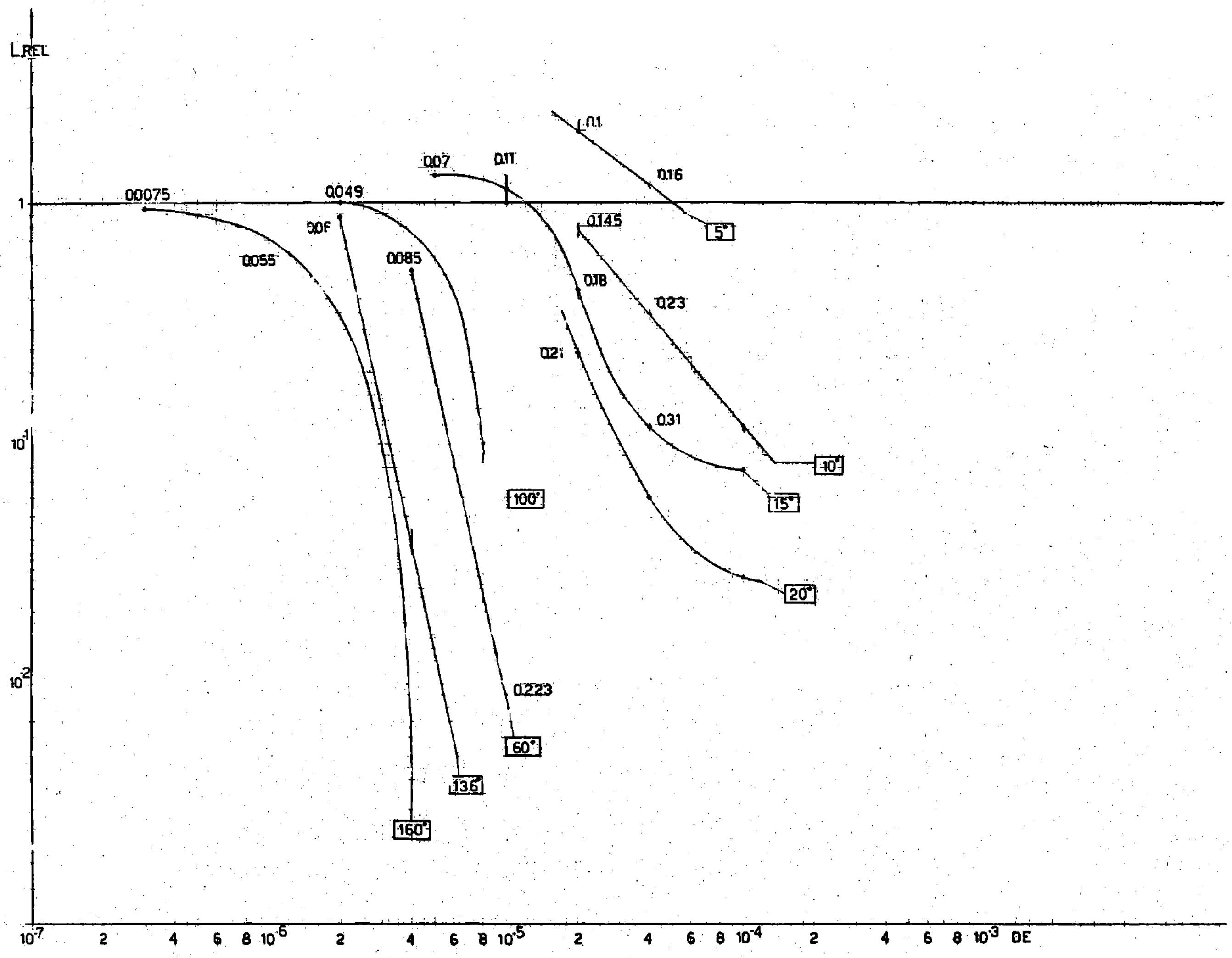

Fig. 1

where DE_+ e DE_- are proportional to the positron and electron currents.

Even in this case, after a certain number of interactions ($> 1/\varepsilon$) the beams reach a steady state and we calculate the mean value of L on somothing like 50 successive interactions.

In the diagram is plotted the quantity $L_{rel} = \bar{L}/L_0$ under the hypothesis $DE_+ = DE_- = DE$ as a function of μ and DE.

Most of these points have been computed with the 7090 of Ispra in 1962 and have required nearly 10 sec per interaction.

Some points have been computed over again at the CDC this year (1,3 sec per interaction) in order to control the influence of ε, $< \bar{z}^2 >^{1/2}$ remaining inchanged, and to study the luminosity fluctuations between interactions.

On every curve of the diagram is marked the value of μ (betatron phase angle between successive interaction regions modulo π) and the ΔQ per interaction, calculated for the natural beam central density as follows:

$$\Delta Q = (1/2\pi)\ \cos^{-1}\left(\cos\mu - \frac{\text{sen}\mu\ DE\ N}{\sqrt{2\pi\bar{z}^2}}\right) - \mu$$

the vertical segment indicates the order of magnitude of the fluctuations; this has been computed only for the interesting cases.

REFERENCES

(1) M. Bassetti: Calcoli numerici sugli effetti di carica spaziale in un anello di accumulazione per elettroni e positroni, Frascati report LNF-62/35 (1962).

(2) The results are also summarized in: Int. Conf. of High Energy Accelerators, Dubna, 1963 (Atomizdat, Moscow, 1964) p. 251.

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